Translate a PDA to a CFG.

1. We say that a PDA $M$ is empty-stack acceptance if: when $M$ accepts a word, the stack is empty (the stack bottom symbol $Z_0$ is popped out). // For this kind of acceptance, we don't care if $M$ is at accepting state at all.

Thm. Every PDA $M$ can be made empty stack-acceptance. That is, for each PDA $M$, we can construct a PDA $M'$ s.t.
\[ L(M) = L(M') \]

\[ L \] denotes stack acceptance.

Proof (easy): When \( M \) accepts an input word \( w \) by reaching an accepting state, and then, we need only keep popping out all symbols in the stack. In this way, the \( M \) now is the \( M' \) with empty stack acceptance.

2. Translate a PDA (with empty-stack acceptance) \( M \) into a Cnf. grammar \( G \).
Conceptually, in the grammar $G$, 

$$[p, A, q] \Rightarrow^{*} q \ y_q$$

where $y_q$ is one non-terminal symbol.
How a grammar generates a word:

\[ S \rightarrow oS1 \lor S \]

Diagram:
```
          o S 4
          /  \  /
         S   /  \  /
        /   /    /
       S 1  S 1
```
Two kinds of instruction in M:

1. \( (q, n) \in \delta(p, a, A) \)
   - At state \( p \), while reading input symbol \( A \) with \( \text{top of stack symbol } A \), \( M \) will switch to state \( q \).
   - Pop the \( A \) out.

We have grammar rule in \( G \):

\[ [p, A, q] \rightarrow a \]
$(p_i, r) \in \Delta(p, a, A)$ with $r = B_1 \ldots B_m$ for $m \geq 0$. 

$\delta$ 

$\gamma$ 

$\beta$ 

$\alpha$ 

$\beta$ 

$\gamma$ 

$[P_i, B_1, P_2] [G_1, B_2, G_2] \ldots [G_{m-1}, B_m, G_m]$
In $G$, we have

$[p, a, q] \rightarrow \alpha \theta[p_1, b_1, q_1] \theta[p_2, b_2, q_2] \cdots \theta[p_m, b_m, q_m]$

where $p, b, q, \ldots, q_m \in \mathcal{Q}$.

(3) Initial rule in $G$:

$s \rightarrow \theta[p_0, z_0, q_1]$

where $q \in \mathcal{Q}$.
Example:

\[ \delta(q_0, 1, 0) = \{(q_1, 1)\} \]

\[ \Rightarrow [q_0, 0, 0, 1] \rightarrow 1 \]

- this is ONE non-terminal symbol.
Example:

\((q_0, 0, z_0) \in S(q_0, 0, z_0)\)

\([q_0, z_0, *_1] \rightarrow 0 [q_0, 0, *_2] [*_2, z_0, *_1]\)

where \(*_1, *_2 \in \mathbb{Q} \)