Complexity of programming is on logic.

1. Every CS student must be able to handle complex logic.

2. Concepts of logic can be seen in alternating quantifiers (math216).

3. Quantifiers: \( \forall, \exists \)

4. Most people can handle two layers of quantifiers:
   For each dog, there is a cat such that if the dog likes Walmart then the cat hates WinCo.
5. CS students can handle three layers of quantifiers:
   For each dog, there is a cat such that for each fish, if the cat likes the fish then the dog hates the fish.

6. Try pumping lemma four layers of quantifiers.
   If you can't do P.I.L. then fail!
Pumping lemma for C.F. languages.
// It's a property for C.F. languages.
∀ a ∈ A c ( ... )

∃ A 4 e ∈ 9 A e L ( ... )
Let \( L \) be an infinite c.f. language. Then, there is a number \( n \) s.t.: for each word \( z \in L \) with \( |z| \geq n \), there is a way to write the \( z \) into 5 parts:

\[
z = uvwx_1y
\]

s.t.

\[
|vwx_1| \leq n
\]

\[
|v| \geq 1
\]

\[
\text{For all } i \geq 0, \quad uv^iwx_1^iy \in L.
\]

\( z \) is a "long word"
We only use P.I.L. to show a language is not c.f.

Example: Show that $L = \{a^n b^n c^n : n \geq 0\}$ is not c.f.

Proof: (use P.I.L.) Assume that $L$ is c.f. Then, let $n$ be the number in the P.I.L. Pick a long word $Z = a^n b^n c^n$ with $|Z| = 3n \geq n$.

F-students do this:
1. Pick $Z = a^{100} b^{100} c^{100}$.
2. Pick $Z = a^n b^n$

Pick a word from $L$ that cancels $Z$. 
For each way that we write $z$ into $5$ parts

$z = uvwx\bar{y}$ with

$|uvwz| \leq n$, $|uvx| \geq n$

We will prove in below that $uv^2wzx^2z^x \notin L$.

Then, the result follows
$z = \overline{\overline{a - a b - b c - c}}$

$z = u v w x y$

$t \leq n$

Since $z$ is of three blocks $a ... a b ... b c ... c$ and the length of $vwx$ is at most $n$, we can see that $vwx$ either is contained in a block or crosses two neighboring blocks. In other words, $vwx$ can **not** contain
three kinds of symbols. Therefore,

\[ uv^2w^2x^2y^2 \text{ can not have all three symbols share the same count}, \ i.e., \ uv^2w^x^2y^2 e L. \]
Hws. last problem.

Hint: Write your proof as:

Proof (use p.h.) Assume the language \( L \) is c.f. and let \( n \) be number in the p.h.

Pick \( z = 0^n 1^n 1^n 0^n = 0^n 1^{2n} 0^n \).

\[
\exists \text{ a million lines!}
\]

Contradiction.